

1 (a) Write Mathematica Command to compute 12! using Do loop, While loop, For Loop

```
fact = 1;
```

```
n = 12;
```

```
Do[fact = fact * k, {k, n}]
```

```
fact
```

```
479001600
```

```
fact = 1;
```

```
n = 12;
```

```
While[n > 0, fact = fact * n; n--]
```

```
fact
```

```
479001600
```

```
For[fact = 1; n = 1, n ≤ 12, n++, fact = n * fact]
```

```
fact
```

```
479001600
```

1 (b) Use Mathematica to verify: (i) $\neg (p \wedge q) = \neg p \vee \neg q$

(ii) $\neg (p \vee q) = \neg p \wedge \neg q$

```
lhs =  $\neg (p \wedge q)$ ;
```

```
rhs =  $\neg p \vee \neg q$ ;
```

```
lhs == rhs;
```

```
LogicalExpand[lhs] == LogicalExpand[rhs]
```

```
True
```

```
lhs =  $\neg (p \vee q)$ ;
```

```
rhs =  $\neg p \wedge \neg q$ ;
```

```
lhs == rhs;
```

```
LogicalExpand[lhs] == LogicalExpand[rhs]
```

```
True
```

2 (a) Write Mathematica code to construct a table showing all positive integers less than 500 omitting those integers divisible by 9. The output should have five numbers in each line

```
l1 = Table[k, {k, 1, 499}];
l2 = Table[9 * k, {k, 1, 500 / 9}];
l3 = Complement[l1, l2];
l4 = Partition[l3, 5];
TableForm[l4, TableAlignments -> Left]
```

1	2	3	4	5
6	7	8	10	11
12	13	14	15	16
17	19	20	21	22
23	24	25	26	28
29	30	31	32	33
34	35	37	38	39
40	41	42	43	44
46	47	48	49	50
51	52	53	55	56
57	58	59	60	61
62	64	65	66	67
68	69	70	71	73
74	75	76	77	78
79	80	82	83	84
85	86	87	88	89
91	92	93	94	95
96	97	98	100	101
102	103	104	105	106
107	109	110	111	112
113	114	115	116	118
119	120	121	122	123
124	125	127	128	129
130	131	132	133	134
136	137	138	139	140
141	142	143	145	146
147	148	149	150	151
152	154	155	156	157
158	159	160	161	163
164	165	166	167	168
169	170	172	173	174
175	176	177	178	179
181	182	183	184	185
186	187	188	190	191
192	193	194	195	196
197	199	200	201	202
203	204	205	206	208
209	210	211	212	213
214	215	217	218	219
220	221	222	223	224
226	227	228	229	230

231	232	233	235	236
237	238	239	240	241
242	244	245	246	247
248	249	250	251	253
254	255	256	257	258
259	260	262	263	264
265	266	267	268	269
271	272	273	274	275
276	277	278	280	281
282	283	284	285	286
287	289	290	291	292
293	294	295	296	298
299	300	301	302	303
304	305	307	308	309
310	311	312	313	314
316	317	318	319	320
321	322	323	325	326
327	328	329	330	331
332	334	335	336	337
338	339	340	341	343
344	345	346	347	348
349	350	352	353	354
355	356	357	358	359
361	362	363	364	365
366	367	368	370	371
372	373	374	375	376
377	379	380	381	382
383	384	385	386	388
389	390	391	392	393
394	395	397	398	399
400	401	402	403	404
406	407	408	409	410
411	412	413	415	416
417	418	419	420	421
422	424	425	426	427
428	429	430	431	433
434	435	436	437	438
439	440	442	443	444
445	446	447	448	449
451	452	453	454	455
456	457	458	460	461
462	463	464	465	466
467	469	470	471	472
473	474	475	476	478
479	480	481	482	483
484	485	487	488	489
490	491	492	493	494

2 (b) Write Mathematica code to find the transformed equation by removing 2nd term of equation equation $4x^3 + 3x^2 - x + 2 = 0$

$$f[x_] = 4x^3 + 3x^2 - x + 2;$$

$$f1[x_] = f[x] / 4$$

$$\frac{1}{4} (2 - x + 3x^2 + 4x^3)$$

$$f1[x] /. x \Rightarrow (y - 1/4) // \text{Simplify}$$

$$\frac{19}{32} - \frac{7y}{16} + y^3$$

3 (a)

$p = (x - 1) * (x - 2)^2 * (x - 3)^3;$

$q = (x - 1)^2 * (x - 2) * (x - 3)^4;$

$a = \text{PolynomialGCD}[p, q]$

$(-3 + x)^3 (-2 + x) (-1 + x)$

$b = \text{PolynomialLCM}[p, q]$

$(-3 + x)^4 (-2 + x)^2 (-1 + x)^2$

$\text{Expand}[a * b] = \text{Expand}[p * q]$

True

3 (b)

$k = 1; \text{set1} = \{ \};$

$\text{While}[\text{Fibonacci}[k] < 600, \text{set1} = \text{Append}[\text{set1}, \text{Fibonacci}[k]]; k++]$

$k = 1; \text{set2} = \{ \};$

$\text{While}[\text{Prime}[k] < 600, \text{set2} = \text{Append}[\text{set2}, \text{Prime}[k]]; k++]$

$\text{Intersection}[\text{set1}, \text{set2}]$

$\{2, 3, 5, 13, 89, 233\}$

5 (a)

5(a)

```

M = ({(2, 1, 1, 8), (1, -1, 2, 1), (1, 1, -1, 4)});
d = M[[All, {1, 2, 3}]];
dx = M[[All, {4, 2, 3}]];
dy = M[[All, {1, 4, 3}]];
dz = M[[All, {1, 2, 4}]];
Print["x=", Det[dx] / Det[d], ", y=", Det[dy] / Det[d], ", z=", Det[dz] / Det[d]]
x=2,y=3,z=1

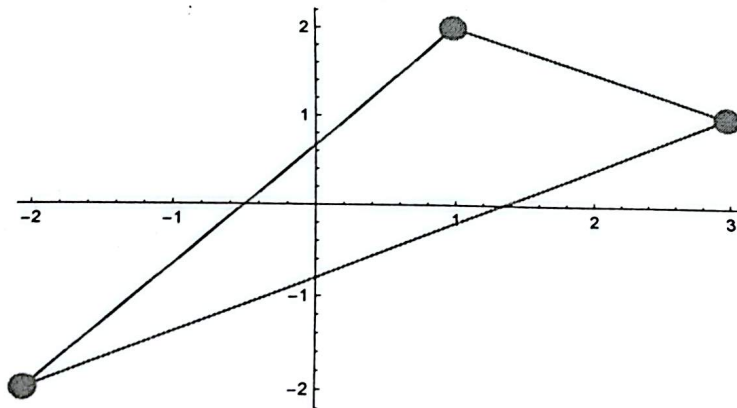
```

5 (b)

```

u = {1, 2};
v = {3, 1};
w = {-2, -2};
pt1 = {u, v, w};
pt2 = {u, v, w, u};
p1 = ListPlot[pt1, PlotStyle -> {Hue[0.56], PointSize[0.04]}];
p2 = ListPlot[pt2, PlotStyle -> {Hue[0.56], PointSize[0.04]}, Joined -> True];
Show[p1, p2]

```



6 (a)

$$ff = 6x^2 + 5xy - 4y^2 + 7x + 13y - 3;$$

$$a = 6; h = 5/2; b = -4; g = 7/2; f = 13/2; c = -3;$$

$$del = a * b * c + 2f * g * h - a * f^2 - b * g^2 - c * h^2$$

$$\text{Simplify}[\{(h * f - b * g) / (a * b - h^2), (h * g - a * f) / (a * b - h^2)\}]$$

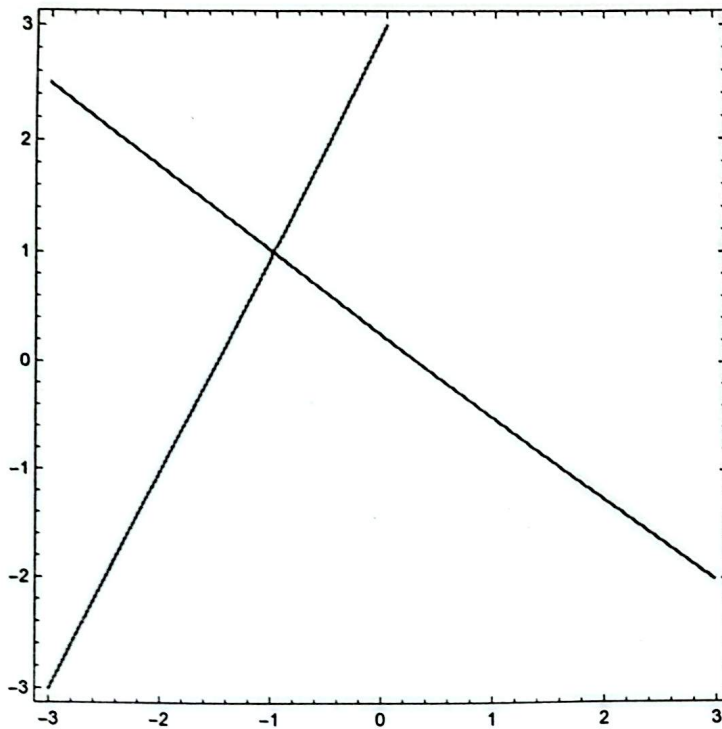
$$\text{ArcTan}\left[\left(2\sqrt{h^2 - a * b}\right) / (a + b)\right]$$

0

{-1, 1}

$$\text{ArcTan}\left[\frac{11}{2}\right]$$

$$\text{ContourPlot}[6x^2 + 5xy - 4y^2 + 7x + 13y - 3 = 0, \{x, -3, 3\}, \{y, -3, 3\}]$$



6 (b) i

$$u = \{u1, u2, u3\};$$

$$v = \{v1, v2, v3\};$$

$$w = \{w1, w2, w3\};$$

$$lhs = \text{Cross}[\text{Cross}[u, v], w] // \text{Expand};$$

$$rhs = v * (u.w) - u * (v.w) // \text{Expand};$$

$$lhs == rhs$$

True

6 (b) ii

6(ii) $a = \{1, 2, -3\};$ $u = \{1, 0, 0\};$ $v = \{0, 1, 0\};$ $w = \{0, 0, 1\};$

$$\text{angle1} = \text{ArcCos} \left[\frac{a \cdot u}{\sqrt{a \cdot a} \sqrt{u \cdot u}} \right] // N$$

1.30025

$$\text{angle2} = \text{ArcCos} \left[\frac{a \cdot v}{\sqrt{a \cdot a} \sqrt{v \cdot v}} \right] // N$$

1.00685

$$\text{angle3} = \text{ArcCos} \left[\frac{a \cdot w}{\sqrt{a \cdot a} \sqrt{w \cdot w}} \right] // N$$

2.50107

7 (a)

$f[x_] = x^2 + x - 2;$

$x1 = 2;$

$y1 = 4;$

$m = D[f[x], x]$

$1 + 2x$

$m1 = m /. \{x \rightarrow x1, y \rightarrow y1\}$

5

$y - y1 == m1 * (x - x1) // \text{Simplify}$

$x - x1 + m1 (y - y1) == 0 // \text{Simplify}$

$5x == 6 + y$

$x + 5y == 22$

7 (b)

$x1 = 1; y1 = -2; z1 = 3; x2 = 2; y2 = -3; z2 = 5;$

$l1 = 2; m1 = 3; n1 = 4; l2 = 3; m2 = 4; n2 = 5;$

$ang = \text{ArcSin}\left[\sqrt{(m1 * n2 - m2 * n1)^2 + (n1 * l2 - n2 * l1)^2 + (l1 * m2 - l2 * m1)^2}\right];$

$l = (m1 * n2 - m2 * n1) / \text{Sin}[ang];$

$m = (n1 * l2 - n2 * l1) / \text{Sin}[ang];$

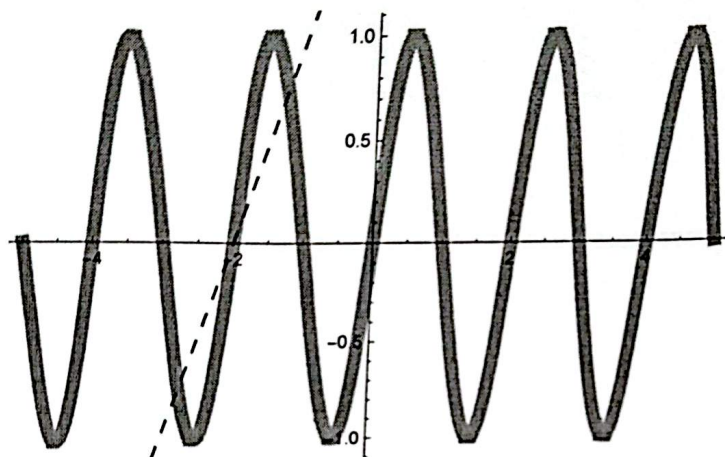
$n = (l1 * m2 - l2 * m1) / \text{Sin}[ang];$

$sd = \text{Abs}[l * (x2 - x1) + m * (y2 - y1) + n * (z2 - z1)] // \text{Simplify}$

$\frac{5}{\sqrt{6}}$

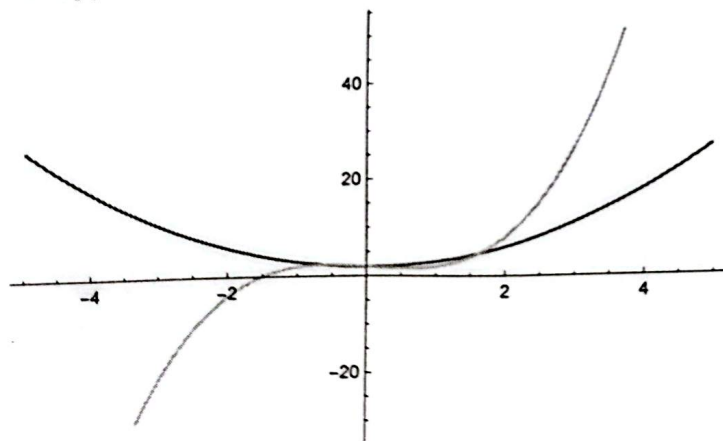
8 (a)

```
f1 = Plot[Sin[ $\pi$  * x], {x, -5, 5},  
  PlotStyle -> {RGBColor[1, 0, 0], Thickness[0.02]}, DisplayFunction -> Identity];  
g1 = Plot[x + 2, {x, -5, 5}, PlotStyle -> {RGBColor[0, 0, 1], Dashing[0.02]},  
  DisplayFunction -> Identity];  
Show[f1, g1]
```



8 (b)

```
Plot[{ $x^2 + 2$ ,  $x^3 - x + 2$ }, {x, -5, 5}, PlotStyle -> {RGBColor[1, 0, 0], RGBColor[0, 1, 0]}]
```

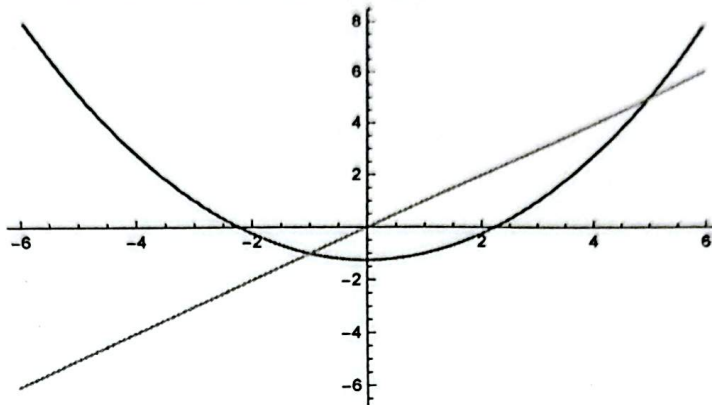


(10 a) .

$$f[y_] = \frac{y^2 - 5}{4};$$

$$g[y_] = y;$$

Plot[{f[y], g[y]}, {y, -6, 6}]



Solve[f[y] == g[y]]

{{y -> -1}, {y -> 5}}

a = -1; b = 5;

Integrate[g[y] - f[y], {y, a, b}]

9

(10 bi) .

$$f[x_, y_, z_] = x^2 + y^2 - z^2;$$

Integrate[f[x, y, z], {x, 0, 1}, {y, -3, 3}, {z, -4, 4}]

-96

(10 bii) .

Solve[x^2 + 16 y^2 == 16, x]

{{x -> -4 Sqrt[1 - y^2]}, {x -> 4 Sqrt[1 - y^2]}}

Solve[x^2 + 16 y^2 == 16, y] /. x -> 0

{{y -> -1}, {y -> 1}}

$$V = \int_{-1}^1 \int_{-4\sqrt{1-y^2}}^{4\sqrt{1-y^2}} \int_0^{x+4} dz \, dx \, dy$$

$$16\pi$$